Material optical constants are of critical importance in optical coatings. They consist of two parameters, refractive index \( n \) and extinction coefficient \( k \), and they are often written together in the form of a complex number \( n - ik \). In fact, as any microwave practitioner will tell us, two more parameters are required for accurate interference structure calculations, the real and imaginary parts of characteristic impedance or admittance. However, in the optical regime we benefit from an immensely important simplification. At the exceedingly high frequencies that characterize light and optics, characteristic admittance is directly proportional to \( n - ik \), the constant of proportionality being \( \varepsilon \), the characteristic admittance of free space, equal to \( 1/376.73031 \) siemens. Thus we need only \( n - ik \) for our calculations. We understand well the concept of optical constants but it has taken more than two millennia for this frequency, \( \omega = 2\pi f \), to become stable enough so that it is a common activity. A true history (we avoid the philosophical question of what this can possibly mean) is beyond complexity and accounts of progress describe individuals to whom are attributed advances in knowledge. Of necessity we must follow that method, but we must not fall into the trap of thinking that these philosophers worked in isolation. They were part of a community that possessed advanced knowledge and a multitude of skills, including the ability to construct accurate instrumentation. Our account largely concerns Europe and the Mediterranean because that is the information that exists.

Refraction is only one aspect of light and its propagation, and, at least in the early development of optics, impossible to disentangle from the whole. Optics in early Greek science is much concerned with the nature of vision. Although vision was

**Snell's Law**

The refractive index, \( n \), first appeared in connection with refraction, that is the deviation of a ray of light when it passes through a surface between two media of differing refractive index. The incident ray is broken into a reflected portion and a transmitted, or refracted, portion, all of the various directions being coplanar with the normal to the surface. It was known at a very early stage that the angle between the normal and the reflected direction was always equal to that between the normal and the incident ray and this is the Law of Reflection. Not as much was understood about the refracted ray beyond the fact that its direction changed and it was a very long time before the Law of Refraction, the Law of Sines, the Law of Descartes, or Snell's Law, as it is variously called, was enunciated and still later when the concept of refractive index appeared. It is still not clear when all this actually occurred nor who really should have the priority.

The law is remarkably simple:

\[
n_0 \sin \theta_0 = n_1 \sin \theta_1 \quad (1)
\]

where \( n_0 \) and \( n_1 \) are the refractive indices on either side of the surface and the angles \( \theta_0 \) and \( \theta_1 \) are the corresponding angles between the ray directions and the surface normal.

**Early Ideas of Refraction**

The optical meaning of the word refraction is very familiar to us, as is the parameter refractive index. The term is derived from a combination of the Latin prefix re together with the third conjugation Latin verb frangere meaning to break, or sometimes to defeat, and with a perfect passive participle of fractus meaning broken.

We do not know where or when our story really starts. Surely as soon as man could make a spear it would have been observed that when partially immersed obliquely in water it would apparently bend at the point of entry. That this was a purely visual effect would have been well established. Apart from effects like this, there would also have been a sense of a line of sight as an apparent straight line.

Our most certain interpretation of history is derived from surviving documents. Without written records we are reduced to induction based on discovered artifacts. It seems clear that the Mesopotamians and Egyptians had considerable optical, geometrical and perhaps even trigonometrical skills, otherwise their advanced constructions would have been impossible, but we have little documentary details. Some worked crystals, shaped as lenses, date back to around 700 BCE. There is much debate about their possible use, but it appears likely that a primary one would have been as a burning glass. Even so, concentration of the sun's rays would have been observed and interpreted as some kind of bending of light in a transparent solid.

**The Greek Philosophers**

Our more certain knowledge, although still fragmentary, starts with the Greek philosophers although the writings that we rely upon are all centuries-later accounts and virtually all copies of copies. Nowadays we tend to interpret the term philosophy rather narrowly, mostly denoting the study of the meaning of existence, reality or morality, but the term has a much larger connotation. Especially in respect of the ancients, the term essentially means the study of all wisdom including what we would now describe as science and technology.

Ancient Greece was not really a country but rather a collection of city states spread over much more of the Mediterranean region than modern Greece and united by a common Greek culture but separated politically. Yet this loose collection of states was responsible for an almost incredible volume and rate of scientific development. Science advances steadily over a broad front as a common activity. A true history (we avoid the philosophical question of what this can possibly mean) is beyond complexity and thus accounts of progress describe individuals to whom are attributed advances in knowledge. Of necessity we must follow that method, but we must not fall into the trap of thinking that these philosophers worked in isolation. They were part of a community that possessed advanced knowledge and a multitude of skills, including the ability to construct accurate instrumentation. Our account largely concerns Europe and the Mediterranean because that is the information that exists.
recognized as connected with light - one cannot see in the dark - it was not necessarily recognized as a simple sensing of light and its distribution. There were two conflicting theories that persisted for many centuries, the extramission theory that held that the eye itself was the source of sight-enabling radiation and the intromission theory that held that sight-enabling radiation emanated from the object itself [1]. Even today there is a common belief that a person can sense the gaze of someone looking at them [1].

Founded originally by Minoans, Miletus, situated on the west coast of southern Turkey, was, in the 7th Century BCE, an important Ionian city. Thales of Miletus (his approximate life span was 624-546 BCE) is one of the earliest Greek philosophers of whom we have a little knowledge, most deduced from later writings by others. He is credited with the introduction of the scientific method by avoiding myth and the supernatural in his studies of natural phenomena, and is reputed to be the inventor of geometry. He was apparently able to measure the height of buildings by recognizing that the ratio of height to shadow in the building was the same as that of his height to his shadow, demonstrating his appreciation of the straight path of a light ray in a homogeneous medium. Thales was clearly an exceptional figure but it does seem unlikely that Thales was responsible for a scientific revolution at all on his own. The later attributions to Thales might better be understood as an assessment of the general scientific climate at that time.

We have slightly better records when we turn to Aristotle (384-322 BCE). Aristotle was born in Macedonia but spent his early years in Athens at Plato's academy. Plato (possibly 428-348 BCE) was an extramission adherent but Aristotle argued for intromission. Since he believed that some kind of motion was what was actually transmitted, he also argued for the existence of the ether to support the movement (not completely different from Young's ideas). He struggled, however, with the nature of an image. He also made use of a camera obscura and surely must have understood that light rays cross each other without any influence of either on the other a principle we owe to Huygens. On the death of Plato, Aristotle returned to Macedonia where he was tutor to Alexander III of Macedonia, known as Alexander the Great, conqueror of the world.

Alexander was an exceptional general, possibly the greatest tactician who ever lived. His most significant achievement, from our current point of view, was the founding in 331 BCE on the northern coast of Egypt, just to the west of the Nile Delta, of Alexandria, the greatest Greek city of the Hellenistic period. Here was established the Musaeum, a center of Greek culture that included a great library and was essentially a university in everything except name.

At the Musaeum, Euclid of Alexandria (his dates are not known but roughly some time in the 4th to some time in the 3rd Centuries BCE) wrote his Elements, used for the teaching of mathematics and especially geometry into the 20th Century, and perhaps in some places even the 21st. He also wrote a treatise on Optics. The original was, of course, in Greek and only later, much later, copies exist, and an English translation has been published by Burton [2]. Many scholars hold that Euclid favored the extramission theory but it is not completely clear from his Optics. We are all familiar with the idea of a line of sight drawn from the eye to the object as a geometrical construction that simplifies analysis but in no way represents emission of light from the eye, and, since the Optics is virtually pure geometry, this may well have been Euclid's intention. Although refraction is not considered, it is clear that the laws of reflection were well understood and Euclid includes an account of the use of a mirror in estimating height when the sun is obscured so that no shadow is present.

A contemporary of Euclid, Epicurus (342-270 BCE) from the island of Samos, just off the Turkish coast, also held an intromission theory but complicated it by attempting to explain image formation as the transmission of a kind of replica film, or skin, that traveled from the object to the eye and contained the complete visual shape of the object.

The Hellenistic city of Syracuse, a kingdom on the east coast of modern Sicily, was home to Archimedes (probably 287-212 BCE) who is justly considered to be one of the greatest mathematicians, possibly the greatest, of all time, but Archimedes was active in every area of science and technology. Unfortunately almost all that we know of him is indirect. The earliest existing manuscript is a copy, dating around the middle of the 10th Century CE, of earlier Greek documents, and is known as the Archimedes Palimpsest. Documents written in Medieval times commonly used animal skin parchment. Writing on such parchment could be scraped off, a process known as palimpsesting, so that the parchment could be used again. The Archimedes Palimpsest is made up of a number of documents, not just of Archimedes, that were scraped, folded in half, reused and rebound into a prayer book sometime in the 13th Century. In 1906, Johan Ludvig Heiberg, a professor of philology at the University of Copenhagen and the leading expert on Archimedes, discovered that the very faint original writings contained seven texts by Archimedes, three of which were hitherto

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unknown. None of these, however, contains anything on optics. For a fascinating account of the recent restoration of the palimpsest see [3].

We know that Archimedes was an accomplished astronomer and later accounts of the use of arrays of burning mirrors as a defensive weapon, confirm that Archimedes was skilled in optics. He had also studied, perhaps under Euclid, or certainly his immediate successors, in Alexandria, and, according to Theon of Alexandria (335–405 CE) had written a Catoptrica, [4], now completely lost. Still later, Olympiodorus the Younger of Alexandria attributed to Archimedes [4] the appearing coin phenomenon, where an out-of-sight coin at the bottom of an opaque container becomes visible with the addition of water. It seems likely that Archimedes wrote on refraction, perhaps the first person to do so.

Previously a supporter of Rome, Syracuse threw its support behind Carthage during the Second Punic War resulting in its attack and subsequent siege by a Roman force commanded by Marcus Claudius Marcellus. Ingenious assault-repulsion engines designed by Archimedes kept the Romans at bay for some two years until overconfidence on the part of the defenders resulted, in 212 BCE, in the penetration of the Roman forces into the major part of the city. Archimedes, now well over 70 years of age, was slain, reputedly by a single Roman soldier, in spite of Marcellus's strict orders that he be spared. The central citadel of the city then held out for a further eight months until finally capitulating.

### The Roman Influence

Having defeated and utterly destroyed Carthage in the Third Punic War, Rome was supreme in the Mediterranean and gradually extended its territory. Alexandria fell under Roman rule in 80 BCE although it had been much influenced by Rome even before that. Then the Roman Republic was replaced by the Empire in 27 BCE with Alexandria and Egypt becoming a Roman province. Alexandria still continued as a center of culture and learning.

Hero of Alexandria (10-70 CE) would certainly be described as an engineer today. He invented a vending machine that provided a measured quantity of holy water when a coin was entered, an automatic system for opening temple doors, probably the first steam engine, and many other applications. He is credited with being the first to enunciate the idea of light's following the shortest path between two points, later developed by Fermat. He wrote on the properties of mirrors. As with the other Greek philosophers his original writings, in Greek, have completely disappeared so that we must rely on much later references.

The bending of the direction of light was clearly known by that time but quantitative measures were still missing. Then Claudius Ptolemaeus (100-170 CE) a Roman citizen of Alexandria, known in more modern terms as Ptolemy and not to be confused with the Ptolemaic rulers of Egypt, wrote his great treatise on optics, in Greek of course, which was still the scientific language. The original version has vanished like the other Greek manuscripts and we must rely on a much later 12th Century version in Latin, translated from Arabic, itself translated from Greek, and missing some parts, and finally, translated only recently into English [5].

The Optics makes it clear that Ptolemaeus was a supporter of the extramission model of vision but this does not affect the validity and accuracy of his measurements of refraction. He knew that the incident, refracted and reflected rays were coplanar with the normal to the surface thus forming the plane of incidence, and he constructed a special very accurate angular scale of bronze so that he could make measurements of the angles. His results on refraction between air and water, between air and glass and even between water and glass for angles of incidence from zero to 80° are almost correct, although the values of index that we can deduce from them are somewhat low. Ptolemy also studied apparent depth showing that this was due to refraction. However he did not arrive at any regular functional relationship connecting incident and refracted angles, although he stated [5] that "the angles do bear a certain consistent quantitative relation to one another with respect to the normals." A good recent analysis is given by Wilk [6]. We note that Ptolemy's measurements on glass required the construction of a glass semi cylinder the flat surface of which contained the cylindrical axis. Considerable skill and knowledge of glass and its working would have been required in this. Ptolemy also realized that refraction in the atmosphere was responsible for displacements of stellar positions near the horizon. An excellent account of the history of atmospheric refraction is given by Lehn and van der Werf [7].

By the time of Ptolemaeus, Rome controlled the entire Mediterranean region. Byzantium, a Greek colony on the Bosphorus from the seventh century BCE, was now Roman, but like the entire eastern side of the Roman Empire, its culture was still Greek and, although Latin was the official language, Greek was the spoken and written language. Constantine the Great (272–337 CE), Roman Emperor from 306 to 337 CE, reorganized and revitalized the Empire. Much of his early career was spent in the east and, as Emperor, he established a new imperial residence at Byzantium that was renamed Constantinople. His preference for Constantinople, an already important cultural and political city, soon rendered it the de facto capital of the Empire. Increasingly, Constantinople took the cultural leadership from Alexandria.

Gradually the Roman Empire split into two halves, the Latin-speaking Western part succumbing to attacks by the Visigoths and Vandals late in the 5th Century but the Greek-speaking Eastern part lasting until the 15th Century with the Greek language's becoming official in the 7th Century. Constantinople was now the most important center of learning, culture and commerce and hosted exceptional activity in the transcribing and preservation of the Greek philosophical manuscripts most of which had been written on papyrus. The Empire in the East became known as the Byzantine Empire in modern writings.

The 6th Century saw Justinian I as emperor, who was the last emperor with Latin as his first language. From then on the history of the Byzantine Empire becomes complex and still more turbulent with one war after another, each changing the political geography of the region and almost impossible to follow. Confusion was increased by the crusades, starting with the first in 1095, which although nominally inspired by religious motives were completely political in reality and consisted of essentially uncontrollably rapacious mixed forces looking largely for plunder. Constantinople was sacked by the Fourth Crusade in 1204 and the Byzantine Empire was split in two, the Nicaean part of the Empire (Nicaea is situated a little south of Constantinople) recovering control of Constantinople in 1261. However, the Byzantine Empire never really recovered from the effects of the Fourth Crusade and was
finally destroyed in a long conflict with the growing Ottoman Empire.

**The Arabs and Ottomans**

What is known as the Golden Age of Arab science [8] began in the middle of the 8th Century with the construction of Baghdad on the site of an older settlement by the Abbasid Caliphate (the third after the Prophet Muhammad). The unifying of the Arab tribes on the Arabian Peninsula in the middle of the 7th Century CE by the Prophet Muhammad launched the Islamic Empire. The expansion of the Islamic world was swift, reaching into India and southern Europe. The teachings of the Prophet strongly emphasized scholarship over martyrdom and welcomed medical research. Baghdad became the most important city of the Islamic Empire and a center of scientific and medical scholarship. The insatiable appetite for knowledge was fed from all possible sources, including the Byzantine Empire, in all possible languages, including Chinese, all of which were translated into Arabic by an army of translators. Research into all aspects of science and medicine was pursued. Southern Spain, ruled by Arab Muslims since 711, competed with Baghdad in the encouragement of science. One of the great achievements of the Empire was the ready availability of books that were written in the common language. The book industry and associated libraries sprang up everywhere. The books mainly used paper rather than parchment, the production of which had been learned (by force) from China. One of the books translated into Arabic was Ptolemy's Optics.

Because of the disturbed history of the Mediterranean our documentary evidence for the Golden Age is rudimentary. But given the drive for knowledge, its availability in the form of books, and the exceptionally favorable political climate, science and technology, and especially for us, optics, flourished, so that rapid development continued. Ptolemy's book was certainly readily available in Arabic and appears to have exerted considerable influence. Once more we are reduced to picking out a few scattered names of which some knowledge has survived.

Burning glasses, of great importance throughout the history of man, were the subject of a treatise written in the 10th Century by a scientist in Baghdad named Ibn Sahl (probably 940–1000) [9]. Roshdi Rashed discovered that two obscure incomplete Arabic documents, one in Tehran, rather dilapidated with pages out of order, and one in Damascus, were actually different fragments of a single document authored by Ibn Sahl. Although pages were still missing, he succeeded in reassembling and then translating the work. This treatise is of great importance for a number of reasons. Science does not flourish in a vacuum. The very existence of the treatise is strong evidence of a scientific community sufficiently advanced in knowledge for the treatise to be of any use. It deals with optics, as distinct from vision and it includes the design of aspheric lenses by a kind of ray tracing. It also contains a geometrical construction that, although in a different form, we can recognize as equivalent to Snell's Law, and is, therefore, the earliest known enunciation of a law of refraction. There is no account of any experimental measurements nor is there any sign of the modern refractive index as a material property. It does suggest

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that a “consistent quantitative relationship” amongst the angles, as suggested by Ptolemy, was, by now, quite well known.

We are more familiar with Ibn al-Haytham (965-1039) known also by his Latin name Alhazen, or sometimes Alhazen, possibly a student of Ibn Sahh, who was author of many books on optics, chief among them being his Kitab al-Manazir or Book of Optics. The book was translated into Latin in 1270 as Opticae Thesaurus Alhazen [10] although there are accounts of an earlier translation in the 12th Century by Gerard of Cremona who worked in Toledo [11]. The Latin version of this book was well known to western scientists and exercised great influence on the further developments of optics. It clearly builds on Ptolemy’s Optics, but that was unknown until quite recently because no version of the Optics had previously appeared to have survived. Alhazen dismissed completely any ideas of extramission and proposed a detailed model of the human eye that presented much that was correct. However he misunderstood the function of the retina and believed that somehow the image in the eye was upright and reduced in extent to fit into the optic nerve so that it could be transmitted to the brain. We have to wait for Kepler’s great leap in understanding [12]. Alhazen was presumably familiar with Ibn Sahh’s work but although refraction is treated extensively in his book there is no account of any experimental work nor any functional Law of Refraction in it.

The Byzantine Empire had taken Sicily in the 6th Century as part of a bid to reunite the split Roman Empire. Sicily was then conquered by Muslim Arabs in the 9th Century and remained in their hands as an essentially independent state until the 11th Century conquest of southern Italy and Sicily by the Normans. Sicily was a rich source of Arabic records amongst which was a copy of the Arabic translation of Ptolemy’s Optics, although missing Book I completely and part of Book V. Eugenius of Palermo (likely 1130 – 1202) was an admiral of the Kingdom of Sicily who was native in Greek and skilled in Arabic and Latin. In or around the 1150’s he translated Ptolemy’s Optics from Arabic to Latin. This incomplete translation is all we have of the Optics and even this itself disappeared, until later copies were rediscovered at the end of the 18th Century. It has since been translated both into French and English [5]. Book V is on refraction and contains Ptolemy’s measurements as well as his experimental procedure.

The Ottoman Empire began in the middle of southern Turkey in the late 13th Century when Osman, ruler of a small principality, declared himself to be a Sultan and established the Ottoman Dynasty. There then followed a continual conflict between what became the Ottoman Empire and the Byzantine Empire, in which the Ottomans gradually increased their territory. From the 14th Century onwards, the Balkans, as the result of a vicious conflict, were dominated by the Ottoman Empire that now extended from the Middle East and that turned its attention towards Constantinople. There followed a long siege of Constantinople by Sultan Mehmet II who overcame the city in 1453 effectively destroying the Byzantine Empire that now gradually faded away. This cemented the Ottoman control of the eastern Mediterranean, but their subsequent defeat at Vienna followed by the failure of their siege of Malta and their loss of the great sea battle of Lepanto, all in the 16th Century, halted the Empire’s advance and began its long decline. It finally ended with the abolition of the Ottoman Sultanate in 1922.

Science and technology were greatly encouraged by the Ottoman rulers under whom Constantinople continued as a center of scientific and technological development, although little written evidence remains. As a result, not a great deal is known of optical developments in the Ottoman Empire. Rather like the Islamic Empire, the details of scientific developments are lost and few names survive. In the 16th Century, the great astronomer in Constantinople, Taqi al-Din ibn Ma’ruf (probably 1521-1585) wrote the Book of Optics [13], which exists today in four copies. Taqi al-Din’s model of light is a transmitted movement having a certain speed in a medium, being less in a denser medium, and responsible for refraction as detailed in Book III of his work. For such a book to have been written there had to exist a surrounding supporting climate of scholarship and knowledge. However it was rather later than the Arabic texts that were translated into Latin and that influenced the Western Europeans, so it was not known there.

Western Europe
Our focus in the 13th Century now moves to Western Europe where the rate of scientific advance was overtaking that in the east. Refraction continued to be an important topic. Although there was gradual improvement in understanding, there was still a long way to go before the appearance of the material parameter we know as the refractive index. Our account will continue in Part 2.

References

3. The fascinating story of the Archimedes Palimpsest is told at: http://www.archimedespalimpsest.org/
12. We will deal with Kepler in more detail in Part 2.

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Angus is a Past President of the Society of Vacuum Coaters. He was born and educated in Scotland. In 1979 he moved to Tucson, AZ, where he is President of Thin Film Center, Inc. and Professor Emeritus of Optical Sciences at the University of Arizona. His best-known publication is Thin-Film Optical Filters, now in its fourth edition. In 2002 he received the Nathaniel H. Segerman Memorial Award from the Society of Vacuum Coaters.
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