The Quarterwave Stack: 3. A Building Block

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Introduction
In part one of this series on the quarterwave stack [1] we dealt with the history of this important structure and then in the second part [2], some of the more important properties. In this third and final part we mainly look at some of the ways in which it is used as a building block in optical coatings. We recall that in its classical form the quarterwave stack is a series of dielectric quarterwaves of alternating high and low index. The principal features of its performance are limited zones of high reflectance separated by regions of low reflectance exhibiting fringes, usually referred to as ripple. We also recall that g is given by \( \lambda / \lambda_0 \) where \( \lambda_0 \) is the reference wavelength that, in this article, will usually be the wavelength for which the layers are quarterwaves.

Peak Reflectance Positions
The quarterwave stack is a rather special member of a class of coatings having designs that consist of a repeated dielectric structure:

- Incident Medium | (Layer sequence) \( ^9 \) | Substrate

The performance of such coatings invariably consists of a series of reflecting zones, where the reflectance increases with increasing \( g \), separated by zones of potentially high transmittance with fringes that remain within constant envelopes and increase in density with \( g \). The reflecting zones occur whenever the total optical thickness of the basic layer sequence is a whole number of halfwaves, unless the sequence is itself an absentee structure. A simple example is the structure:

- Air | [0.2(H LL HHH LLLL)] \( 100 \) | Glass

where the power of 100 is chosen to make the envelopes clearly visible and the factor 0.2 is to be applied to all layers in the system. The structure to be repeated is often called a period, and this period will be a single halfwave thick at unity \( g \) and so the potential peak positions are \( g \)-values of 1, 2, 3, 4, 5… However, at \( g = 5 \) all the layers are quarterwaves. An LL layer is therefore a halfwave and an absentee layer. At that value of \( g \) the structure is, therefore, equivalent to HHHH that is itself an absentee structure. A simple example is the structure:

- Air | [0.2(H LL HHH LLLL)] \( 100 \) | Glass

The repeated period is absentee at \( g = 4, g = 8, g = 12, g = 16 \) and so forth. This behavior can be very useful in the design of broad bandpass filters and also of broad reflecting structures. However it can also have some unfortunate consequences as explained in the next section.

The Halfwave Hole
The quarterwave stack as a longwave pass filter has been described in the previous article in this series [2]. The primary problem is the pronounced ripple that must be removed from the pass region, readily accomplished by creating two matching structures between the quarterwave assembly and the surrounding media. A similar procedure yields a shortwave pass filter, Figure 3, where the layers have indices of 1.45 and 2.10 without dispersion and representing roughly silica and tantala. 1000nm is the reference wavelength at which all the core layers are quarterwaves. The potential second-order peak at 500nm is missing because the core layers are exact halfwaves at that wavelength.

It is vital for the suppression of a zone of high reflectance that both high index and low index layers should be halfwaves or multiple halfwaves at precisely the same wavelength. This condition can be perturbed in various ways. In Figure 4 we illustrate what happens when the high-index layers are a little thinner in optical thickness than the low-index layers. The result is the appearance of a narrow peak of higher reflectance and lower transmittance. The effect is quite local and the primary characteristic is virtually unchanged. The narrow peak is known as a halfwave hole and, although we show the effect at a second-order peak position, it can occur at any peak missing because of a halfwave condition.

Such detuning of layer thicknesses might be due to a uniformity problem, or dispersion in the materials of layers that are accurately quarterwaves at the reference wavelength. It can also be due to tilting.
of a coating perfect at normal incidence. The simple way of avoiding the effect is to make sure that the layers are halfwaves at the offending wavelength rather than quarterwaves at the reference wavelength. This, however, does not prevent the appearance of the halfwave hole on tilting. The complete solution of the problem requires an antireflection coating at each and every interface between the quarterwave layers. This antireflection coating should be narrow enough in its characteristic so that it has little effect at the primary wavelength. A two-layer V-coat involving thin layers of the high-index and low-index materials already used in the design, is a useful approach. The ultimate, removing all higher order peaks, is an inhomogeneous layer that transforms the quarterwave stack into a rugate structure [3].

**Extended Zone Reflectors**

With existing materials, the quarterwave stack cannot cover the entire visible region. The usual and simple solution is to place one or two additional quarterwave stacks over the first with staggered reference wavelengths so that together they completely cover the required region. There are two important tricks associated with this.

Simply placing one quarterwave stack over another risks the appearance of a deep and narrow minimum of reflectance in the middle of the desired zone of high reflectance. The reason is that the layers of one stack combine with those of the other to form a set of halfwave layers that turn the assembly into a giant absentee structure. To avoid this it is sufficient to insert a layer of the alternate index and of thickness a quarterwave at roughly the offending wavelength. The design of a coating employing two quarterwave stacks in series then becomes:

\[
\text{Air} \mid (HL)'H\ L'' (H'L')'H' \mid \text{Substrate}
\]

where the primes indicated different reference wavelengths. Without the \( L'' \) layer, there would be an \( HH' \) combination representing a halfwave at an intermediate wavelength. The surrounding \( L \) and \( L' \) layers would represent another halfwave at the identical wavelength, and so on. Since the values of \( p \) and \( q \) will normally be similar the structure would then exhibit a deep narrow fringe of low reflectance. When three stacks are involved, each must be separated from the adjoining by a similar quarterwave of alternate index, and so on.

The second trick is a little more complicated. Over part of the range the light has to pass through the outer stack to reach the inner where it is reflected back out again. This implies passing through the outer stack twice. Absorption has a more serious effect on transmittance than reflectance and so we want the double traversal to occur where the absorbance of the outer stack is at a minimum. In the visible region this implies that the outermost stack should be the thinnest and the deepest stack the thickest because absorbance generally increases towards shorter wavelengths. Permissible levels of absorption in a single quarterwave stack reflector are much greater than in an extended zone reflector and it can sometimes be a little surprising to find how badly materials, absolutely satisfactory in a single quarterwave stack, can behave in an extended zone reflector. The problem is illustrated in the, admittedly continued on page 24.

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**Figure 3.** A shortwave pass filter based on a 41-layer quarterwave stack with reference wavelength 1000nm and with seven layers on either side modified to form matching structures that reduce the ripple.

**Figure 4.** High-index layers that are 3% too thin, and low-index 3% too thick, in the design of Figure 3 cause the appearance of the narrow peak of high reflectance known as a halfwave hole. The performance (orange) is plotted over that of Figure 2 (black). Except at the halfwave hole, there is virtually no difference.
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rather extreme, example of Figure 5 and Figure 6. Figure 5 shows the optical constants of a high index material used in the design of the extended zone high reflectance coating. The low index layer was typical of silica with zero extinction coefficient. Figure 6 shows the calculated performance for the correct sequence, that is thinnest layers outermost, together with that of the opposite, and incorrect, sequence. One small point is that extended zone reflectors should always be tested in reflection. Low transmittance over the extended zone does not necessarily imply high reflectance, Figure 6.

There are other, perhaps somewhat unexpected, problems with extended zone reflectors. The straightforward quarterwave stack with a final high-index layer next to the incident medium exhibits a phase shift on reflection showing a quite slow variation with wavelength on either side of 180° [2]. This implies low sensitivity to contamination [5] and reduced wavefront errors in the presence of poor uniformity [6]. The situation is quite different when an extended zone reflector is concerned. The phase change on reflection varies rapidly in the regions where the light penetrates deeply into the coating. With high reflectance and a phase change on reflection close to zero, the sensitivity to contamination is at a maximum [5], Figure 7. The rapid changes in reflectance phase change also imply enhanced sensitivity of the phase to errors in uniformity that can cause curious wavefront distortion in reflected light. For example, concave errors in uniformity can even be transformed into much larger wavefront errors consistent with a convex uniformity error, and vice versa [6].

Figure 5. Optical constants of the high-index layer used in the extended zone reflector of Figure 6. The low index material is silica with zero extinction coefficient.

Figure 6. The performance of an extended zone reflector consisting of three superimposed quarterwave stacks with inserted decoupling layers as explained in the text. Since the ripple in the outer stacks usually affects the reflectance slightly, the resulting design was then gently refined to smooth the reflectance. The blue curves show the performance of the correct sequence with thinner layers outermost while the red curves illustrate the effect of reversing this sequence so that the thicker layers are outermost.

Figure 7. The contamination sensitivity of the extended zone reflector (correct sequence) of Figure 6 compared with that of a quarterwave stack. This is broadly similar to a curve shown in [5]. Note that on this scale a perfect antireflection coating has a sensitivity of unity.

Bandpass Filters

In its usual form, the matrix method for performance calculation involves the multiplication of a series of square four-element matrices, each representing a single layer. A symmetrical arrangement of films yields a resultant product matrix that can be manipulated into the form of a matrix of a single film with an equivalent phase thickness and characteristic admittance. Although this is true for any films, including metals, straightforward interpretation of the equivalent parameters requires dielectric layers. For dielectric layers, the equivalent phase thickness and equivalent admittance exhibit regions where either both are real or both are imaginary. It was Ivan Epstein who realized that this curious property could be turned into a valid and powerful design technique for edge filters [7]. Regions where both parameters are real correspond to potential pass bands, and regions where they are both imaginary, to stop bands. There are no regions where one is imaginary and the other real. A repeated symmetrical structure is then equivalent to an appropriately thicker single layer with the identical equivalent admittance. This increases the reflectance in the stop bands and the density of the ripple in the pass bands but does not change the ripple envelopes. The ripple can be understood as a simple consequence of a mismatch with the surrounding media. Basic structures much used in the technique because their parameters can be arranged reasonably to match the surrounding media with no additional changes are either 0.5H L 0.5H or 0.5L H 0.5L, both repeated multiple times to yield the appropriate level of rejection. This straightforward application of the method leads directly to the technique, mentioned in part 1 of this series [1], of reducing ripple by starting and terminating a quarterwave stack with eighth-wave layers.

Alfred Thelen further developed the method and extended it to the design of narrow bandpass filters [8]. We start with a quarterwave stack arranged to start and terminate with a quarterwave of the same material so that it is symmetrical. Such a structure shows a narrow band of real, that is dielectric, behavior at the reference wavelength surrounded by broad imaginary regions extending to the boundaries of the high reflectance zone. The equivalent phase thickness at the reference wavelength, where all the layers are quarterwaves, is equal to the total phase thickness of the layers while the equivalent admittance is given...
by the product of the admittances of the odd layers divided by the product of the even ones. The enormous mismatch that this normally implies, is the reason for the high reflectance exhibited at the reference wavelength, in spite of the existence of the potential pass band. To activate the potential pass band requires matching structures on either side. These can conveniently consist of quarterwave layers. We look at a simple example:

\[ (LHLHLHLH \ L \ HLHLHLH) \]

where we have inserted a couple of spaces so that the central L layer is easily identified. We can assume silica and tantala as the two materials with indices taken as 1.45 and 2.15 respectively. There are nine odd L layers and eight even H and so the equivalent admittance is \( y_L^{18}/y_H^{16} \). An estimate of the bandwidth of the filter is a little more difficult. Thelen gives an approximate formula based on the edges of the real region rather than any value of transmittance, but computers are nowadays so fast that the simplest and quickest way to arrive at a value for bandwidth is to calculate the performance of the filter. The bandwidth can be adjusted to some extent but given the materials the discrete nature of the thin film structure implies that the adjustment will be discontinuous. Adding additional quarterwaves to the structure, keeping it symmetrical, reduces the bandwidth. Adding halfwave layers to the structure, again provided it remains symmetrical, will not alter the equivalent admittance but it will increase the total thickness and hence also reduce the bandwidth. However this second modification is weaker than that induced by adding quarterwaves and so we have a type of fine-coarse control over bandwidth. Of course, if we were able to accommodate a continuous variation in layer thicknesses away from quarterwaves then we could have a smoother adjustment but at the cost of a formidable manufacturing task.

Let us return to our filter structure. At present, even though it terminates on an L layer, its passband presents the high reflectance that we associate with our quarterwave stacks. To turn it into a useful filter we need to add the necessary matching structures. With the aid of the quarterwave rule [9] we can see that a structure:

\[ HLHLHLH \]

added in front and

\[ LHLHLHLH \]

behind will transform the admittance presented to the outside world to \( y_L^{18} \). The resulting design is then:

\[ \text{Incident} \mid HLHLHLH \ (LHLHLHLH \ L \ HLHLHLHL) \]

\[ LHLHLHLH \mid \text{Substrate} \]

To complete the design we need to match the entire structure to the incident medium and to the substrate. We recall that the structure appears at the reference wavelength as a slab of low-index material. With a normal substrate of glass or silica and a low-index layer of silica the matching to the substrate is already good enough. The incident medium we can assume as air and then, since the bandwidth of the structure is quite small, a simple two-layer V-coat at the front will suffice. The design is then:

\[ \text{Air} \mid L’ H’ HLHLHLHLH \ (HLHLHLHLH \ LHLHLHLH) \]

\[ LHLHLHLH \mid \text{Glass} \]

where \( L’ H’ \) is the V-coat, \( L’ \) a little greater than a quarterwave and \( H’ \) rather less.

With \( q \) unity this can be seen to be a two-cavity filter. With \( q \) set to two it is a three-cavity, Figure 8, and so on. This usually works well for two or three cavities but with increasing numbers of cavities the increasing thickness of the filter implies denser and denser ripple. There is some dispersion in the equivalent passband properties that implies continued on page 26

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The symmetrical quarterwave stack that we have used in our multiple-cavity filters has still more uses, one being in the design of filters with multiple passbands. The equivalent phase thickness of our 17-layer symmetrical quarterwave stack of the previous section is shown in Figure 9. At the start of the real region, that is at the start of the potential passband, it is \((n+1)/2\); at \(\lambda_0\) it is \(n\pi/2\); at the termination of the real region it is \((n-1)/2\); where \(n\) is the number of quarterwaves in the stack. This pattern applies to any number of quarterwaves arranged in a symmetrical fashion. The values at the start and termination of the pass region suggest that at these wavelengths the symmetrical arrangement, when placed on a substrate might act as a halfwave layer. Dielectric halfwave layers have no effect on either reflectance or transmittance when placed on a substrate might act as a halfwave layer. Dielectric layers, and are sometimes called absentee layers. Thus we might expect to see two deep minima on either side of the reference wavelength in a normal quarterwave stack reflector – and, of course, we know we never see that. The reason is that at these special wavelengths at the zone edge, the equivalent admittance either falls to zero, or rises to infinity. If we examine the product matrix we will see that only one of the off-diagonal terms is zero. Both should be zero for a true halfwave film. The outer edges of the potential pass region are strictly the start of the high reflectance zone. Within the basic symmetrical structure, Figure 9 shows us that there are no wavelengths where the equivalent phase thickness corresponds to a halfwave and, therefore, to an absentee layer. What happens if we place two of these structures in series?

\[(LHLHLHLHLHLHLHL)^2\]

Now the total phase thickness runs from \(16\pi\) to \(18\pi\) and in the very center of the zone, at the reference wavelength the phase thickness is \(17\pi\), that is an absentee. The result is a single-cavity narrowband filter, Figure 10, sometimes called a Fabry-Perot because its structure is similar to a Fabry-Perot etalon.

The bandwidth of the resulting filter is quite small and can be broadened by removing some layers from the quarterwave stack structures, keeping them symmetrical. Let us reduce the number of quarterwaves to thirteen but at the same time increase the number of repeats to three. Now the basic structure has an equivalent phase thickness that runs from \(6\pi\) to \(7\pi\) and for the repeated structure from \(18\pi\) to \(21\pi\). This contains two thicknesses, \(19\pi\) and \(20\pi\) where the structure is an absentee and therefore exhibits a transmission peak. The precise position of the peaks will depend on the bandwidth of the basic structure, which in turn depends on the number of layers. This can be used to tune their position and, as in the normal filter a fine-coarse control can be included by the judicious use of additional halfwaves. Like the basic single-cavity filter, the width of the pass bands is a function of the reflectance on either side of the cavity. Once the positions are established, additional layers can be added outside the basic structure to increase or decrease the reflectance. Figure 11 shows the performance of three structures based on the thirteen quarterwave stack.

Figure 8. The three-cavity version of the narrowband filter.

Multiple Peaks

Figure 9. The equivalent phase thickness of the symmetrical quarterwave stack: \(LHLHLHLHLHLHLHL\). At \(\lambda_0\) it is \(8.5\pi\) and at the edges of the real region \(9\pi\) and \(8\pi\). The symmetrical quarterwave stack that we have used in our multiple-cavity filters has still more uses, one being in the design of filters with multiple passbands. The equivalent phase thickness of our 17-layer symmetrical quarterwave stack of the previous section is shown in Figure 9. At the start of the real region, that is at the start of the potential passband, it is \((n+1)/2\); at \(\lambda_0\) it is \(n\pi/2\); at the termination of the real region it is \((n-1)/2\); where \(n\) is the number of quarterwaves in the stack. This pattern applies to any number of quarterwaves arranged in a symmetrical fashion. The values at the start and termination of the pass region suggest that at these wavelengths the symmetrical arrangement, when placed on a substrate might act as a halfwave layer. Dielectric halfwave layers have no effect on either reflectance or transmittance and are sometimes called absentee layers. Thus we might expect to see two deep minima on either side of the reference wavelength in a normal quarterwave stack reflector – and, of course, we know we never see that. The reason is that at these special wavelengths at the zone edge, the equivalent admittance either falls to zero, or rises to infinity. If we examine the product matrix we will see that only one of the off-diagonal terms is zero. Both should be zero for a true halfwave film. The outer edges of the potential pass region are strictly the start of the high reflectance zone. Within the basic symmetrical structure, Figure 9 shows us that there are no wavelengths where the equivalent phase thickness corresponds to a halfwave and, therefore, to an absentee layer. What happens if we place two of these structures in series?

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Figure 10. The single-cavity filter resulting from a doubled symmetrical quarterwave stack. Bandwidth at the half-peak points is \(0.92\text{nm}\).

Figure 11. Three different versions of the three times repeated 13-layer quarterwave stack.
At this stage the passband shape is similar to that of a single-cavity filter. It can be manipulated into the more rectangular shape of a multiple-cavity filter by repeating the basic structure with the usual inserted coupling layers. Figure 12 shows the effect of a three-times repeat. In this case the coupling layer is of high index because the primary structure terminates with L layers. Since there are two coupling layers they are close to a halfwave absentee at the two peaks. The entire structure is, therefore, effectively absentee at the two peaks leaving the effect of the mismatch between glass substrate and air incident medium. This is eliminated with a simple V-coat.

This example has been of a double-peak filter. Four repeats of the symmetrical quarterwave stack will give three transmission peaks that can be treated in a similar way. Five repeats give four peaks and so on.

The Quarterwave Stack at Oblique Incidence
So far we have concentrated on normal incidence. There are many additional applications of the quarterwave stack at oblique incidence. For example, it is possible to construct polarizing elements [11]. Tilting the quarterwave stack weakens the p-polarized performance while it strengthens the s-polarized performance. This results in a region at the edge of the high-reflectance zone where high transmittance for p-polarization is matched by high reflectance for s-polarization. Over a limited region this results in a useful polarizer. Increasing the angle of incidence still further by immersing the structure in glass yields a still broader polarizer.

Conclusion
Although the quarterwave stack is one of our oldest interference structures it remains one of our most useful and versatile.

References

Angus Macleod is a Past President of the Society of Vacuum Coaters. He was born and educated in Scotland. In 1979 he moved to Tucson, AZ, where he is President of Thin Film Center, Inc. and Professor Emeritus of Optical Sciences at the University of Arizona. His best-known publication is Thin-Film Optical Filters, now in its fourth edition. In 2002 he received the Nathaniel H. Sugerman Memorial Award from the Society of Vacuum Coaters.

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